

Total = 40

1. Answer the following questions, showing your reasoning.

a) How many 3 digit numbers are divisible by 2?

that's the sequence $\{100, 102, 104, \dots, 998\}$

$$a_n = a_1 + (n-1)d$$

$$998 = 100 + (n-1) \cdot 2$$

$$898 = (n-1) \cdot 2$$

$$449 = n-1$$

$$n = 450$$

b) How many 3-digit numbers are divisible by 5?

that's the sequence $\{100, 105, 110, \dots, 995\}$

$$a_n = a_1 + (n-1)d$$

$$995 = 100 + (n-1) \cdot 5$$

$$895 = (n-1) \cdot 5$$

$$179 = n-1$$

$$n = 180$$

c) How many 3-digit numbers are divisible by 2 and 5?

that's the sequence $\{100, 110, 120, \dots, 990\}$

either calculate as before or notice this will be the same as the number

of terms in the sequence $\{10, 11, 12, \dots, 99\}$

$$\text{which is } n = \text{last} - \text{first} + 1$$

$$= 99 - 10 + 1 = 90$$

d) How many 3-digit numbers are divisible by 2 or 5?

$$n(\text{div by } 2 \text{ or } 5) = n(\text{div by } 2) + n(\text{div by } 5) - n(\text{div by } 2 \text{ and } 5)$$

$$= 450 + 180 - 90$$

$$n = 540$$

3 digits

or

— — —	↑	— — —	↑	— — —	↑
1-9		0-9		0,2,4,6,8	
so nine choices		so ten choices		so five choices	

$$n = 9 \cdot 10 \cdot 5 = 450$$

3 digits

or

— — —	↑	— — —	↑	— — —	↑
1-9		0-9		0,5	

$$n = 9 \cdot 10 \cdot 2 = 180$$

or

3 digits

— — —	↑	— — —	↑	— — —	↑
1-9		0-9		0	

$$n = 9 \cdot 10 \cdot 1 = 90$$

or

3 digits

— — —	↑	— — —	↑	— — —	↑
1-9		0-9		0,2,4,5,6,8	

$$n = 9 \cdot 10 \cdot 6$$

$$= 540$$

3. Passwords on a particular computer system consist of Russian alphabet, which contains 31 letters (including the letter "π"). If no letters can be repeated, find the following:

a) the number of possible 3-letter codes

$$\frac{31}{\text{any}} \frac{30}{\text{any}} \frac{29}{\text{any}} = 26970$$

no repetition

(or P_3^{31} if you wish)

↑
order matters

①

b) the number of possible 3-letter codes that begin with the letter "π"

$$\frac{1}{\pi} \frac{30}{\text{any but } \pi} \frac{29}{\text{any}} = 870$$

(or P_2^{30} if you wish)

②

c) the number of possible 3-letter codes that do not begin with the letter "π"

$$\begin{aligned} \# \text{ legal} &= \text{total} - \text{illegal} \\ &= 26970 - 870 = 26100 \end{aligned}$$

(or $P_3^{31} - P_2^{30}$)

②

d) the number of possible 3-letter codes that contain the letter "π" somewhere

$$\# \text{ that do not contain } \pi: \quad \frac{30}{\text{any}} \frac{29}{\text{any}} \frac{28}{\text{any}} = 24360$$

$$\begin{aligned} \# \text{ legal} &= \text{total} - \text{illegal} \\ &= 26970 - 24360 \\ &= 2610 \end{aligned}$$

$$\begin{aligned} \text{or } \# \text{ total} &= \#(\pi _ _) + \#(_ \pi _) \\ &\quad + \#(_ _ \pi) \\ &= 3 \times 870 \\ &= 2610 \end{aligned}$$

②