Section 1.2: Subsets

Definitions

If every element in set A is also a member of set B, then we say that set A is a subset of set B,

$$A \subseteq B.$$

If every element in set A is also a member of set B **and** there is at least one element in B that is not in A, then A is a proper subset of B, and

$$A \subset B.$$

As before, the addition of a slash to the symbol negates it, so $\not\subseteq$ means "is not a subset of" and $\not\subset$ means "is not a proper subset of".

If you like, you can think of the subset and proper subset symbols in the same way that you think of $<$ and $\leq$ symbols: if there is part of an equals bar, then the two sides can be the same, but if the bar is missing, then the right side has to be "larger" in that it has an extra element that the left side does not have.

Example

Let X = {1, 2, 3} and Y = {1, 2, 3, 4}. Are the following statements true or false?

$X \subseteq Y$

$Y \subset X$

$X \subset X$

$Y \subseteq Y$

Answer:

$X \subseteq Y$: T, every element of X is in Y.

$Y \subset X$: F, because 4 is in Y but not in X.

$X \subset X$: F, because there isn't an element in the X on the right hand side that isn't in the X on the left hand side.

$Y \subseteq Y$: T, because every element in the Y on the left hand side is in the Y on the right hand side.

Remember: to show that set A is not a subset of B, you just have to list a single example of an element in A that's not in B. You don't have to list them all. And if you can't find a counterexample, that means that A must be a subset of B.

Example

> True or false? For every set A, $\varnothing \subseteq A$.
>
> Answer: True. No matter what A is, there's no element in $\varnothing$ that's not also in A. Note also that $\varnothing$ is a proper subset of every set except itself.

Subset in Python

The notation for subsets is a little clunky in Python as you can see in the following figure.

```
>>> A={1,2,3}
>>> B={2,3,4}
>>> A.issubset(B)
False
>>>
```

Figure 1: Subsets in Python