Math 173 – Quiz #2

February 5, 2016 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 40 points

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1. For $f(x) = x^2 - 1$ and $g(x) = \dfrac{4}{\sqrt{x}}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. (4 points)

$(f \circ g)(x) = f(g(x)) = \left(\dfrac{4}{\sqrt{x}}\right)^2 - 1$

$= \dfrac{16}{x} - 1$

$(g \circ f)(x) = g(f(x)) = \dfrac{4}{\sqrt{x^2 - 1}}$

$(f \circ g)(x) = \dfrac{16}{x} - 1$

$(g \circ f)(x) = \dfrac{4}{\sqrt{x^2 - 1}}$

note: $\sqrt{x^2 - 1} \neq x - 1$ ack!

mix up $g \circ f$ and $f \circ g$ -1

2. Rewrite the equation of the following parabola in the form $f(x) = a(x-h)^2 + k$ **by completing the square** and state the equation of the axis of symmetry and the coordinates of the vertex. Is the vertex at the maximum or minimum point in the parabola (circle one)? (6 points)

$f(x) = 2x^2 - 12x + 23$

$$\left.\begin{aligned} &= 2(x^2 - 6x + \_\_) + 23 - \_\_ \\ &= 2(x^2 - 6x + 9) + 23 - 2(9) \\ &= 2(x-3)^2 + 23 - 18 \end{aligned}\right\} \text{work } 2$$

$= 2(x-3)^2 + 5$

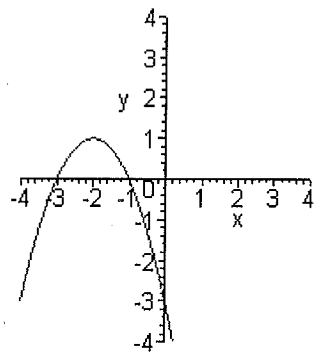
equation: $f(x) = 2(x-3)^2 + 5$ 1

vertex: $(3, 5)$ 1

axis of symmetry: $x = 3$ 1

maximum / minimum 1

3. Write the equation for the function given in the graph below. (The graph hasn't been stretched, just shifted and reflected.) (3 points)



this graph is $f(x) = x^2$ but reflected in x and shifted left by 2 and up by 1:

$$\boxed{f(x) = -(x+2)^2 + 1}$$

4. Is the function *f(x)* even, odd, or neither? Show your work. (2 points)

$f(x) = x^3\sqrt{1-x^2}$ odd

$f(-x) = (-x)^3 \sqrt{1-(-x)^2}$

$= -x^3 \sqrt{1-x^2} = -f(x)$ $\therefore$ odd

5. Factor the following polynomial into linear factors. Then state the zeros of this function and their multiplicities. (5 points)

$f(x) = x^3 + 3x^2 + 7x + 5$

1

1 $f(x) = (x+1)(x+1+2i)(x+1-2i)$

1 $x = -1, -1 \pm 2i$ 1

each with mult 1

$\frac{p}{q} = \frac{\pm 1, \pm 5}{\pm 1} = \pm 1, \pm 5$ ½

none of the positive ones will work

1 $f(-1) = -1 + 3 - 7 + 5 = 0$ ½

so $(x+1)$ is a factor

1 1

$$\begin{array}{r} x^2 + 2x + 5 \\ x+1 \overline{) x^3 + 3x^2 + 7x + 5} \\ \underline{x^3 + x^2 \qquad\qquad} \\ 2x^2 + 7x \quad \\ \underline{2x^2 + 2x \quad} \\ 5x + 5 \end{array}$$

1 $x^2 + 2x + 5$ doesn't factor

→ quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4 - 20}}{2}$

$= \frac{-2 \pm \sqrt{-16}}{2}$

1

$= \frac{-2 \pm 4i}{2} = -1 \pm 2i$

6. Consider the following rational function: (10 points)

$$f(x)=\frac{x-1}{x^2-2x-3}=\frac{x-1}{(x+1)(x-3)}$$

a) What is the y-intercept? set $x=0$

$f(0)=\frac{-1}{1\cdot(-3)}=\frac{1}{3}$ $\boxed{(0,\frac{1}{3})}$ 1

b) What are the x-intercepts? set num $=0$

$x-1=0$ so $x=1$ $\boxed{(1,0)}$ 1

c) Are there any vertical asymptotes? If so, where?

$(x+1)(x-3)=0$ $\boxed{x=-1 \text{ and } x=3}$ 2

d) Are there any horizontal asymptotes? If so, where?

deg num < deg denom $\boxed{y=0}$ 1

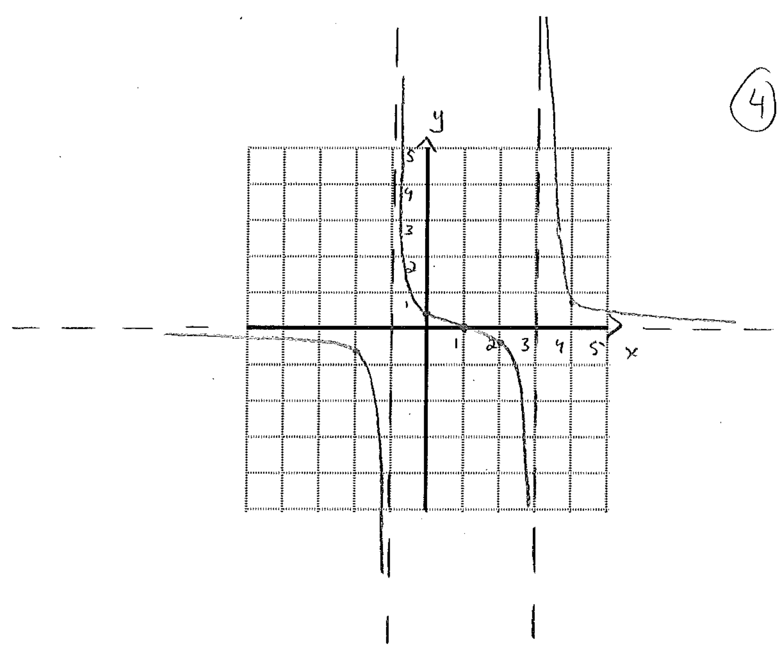
e) Are there any oblique/slant asymptotes? If so, where?

no 1

f) Sketch the graph as accurately as possible.

extra points:

x	y
–2	–3/5
2	–1/3
4	3/5



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7. Use the Rational Zeros Theorem to list all possible rational zeros of *f(x)*. (2 points)

$f(x) = 4x^5 - 2x^4 + 5x + 2$ $\underline{\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}}$

$\frac{p}{q} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

8. Using Descartes' Rule, how many positive real zeros and negative real zeros can the following polynomial have? Do not solve it! (2 points)

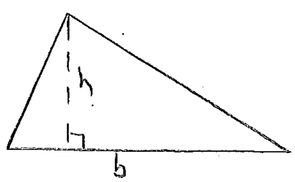
$f(x) = -3x^5 + x^3 - 9$

positive real zeros: $\underline{2, 0}$

negative real zeros: $\underline{1}$

$f(-x) = 3x^5 - x^3 - 9$

9. The sum of the base and the height of a triangle is ~~32~~ 20 cm. Find the maximum area for this triangle, and also state the dimensions of the triangle which give this maximum area. (6 points)



$b + h = 20$

$b = 20 - h$ (1)

$A = \frac{1}{2} b h$ (1)

$= \frac{1}{2}(20 - h) h$

$= 10h - \frac{1}{2}h^2 = -\frac{1}{2}h^2 + 10h$ (1)

finding vertex

method #1:

$h_{max} = -b/2a$

$= -10/2(-\frac{1}{2})$

(1) $= +10$ cm

so $h_{max} = 20 - h = 10$ (1)

$A_{max} = \frac{1}{2} b h$

$= \frac{1}{2}(10)(10)$

$= 50 \text{ cm}^2$ (1)

method #2:

(2) $A = -\frac{1}{2}(h^2 - 20h)$

$= -\frac{1}{2}(h^2 - 20h + 100) - 100(-\frac{1}{2})$

$= -\frac{1}{2}(h - 10)^2 + 50$

so $h_{max} = 10$ cm and $A_{max} = 50 \text{ cm}^2$ (1)

The triangle's base and height are both 10 cm, giving an area of 50 cm^2.