Math 173 – Quiz #3

February 26, 2016 **Name:** Solution Set

Instructor: Patricia Wrean

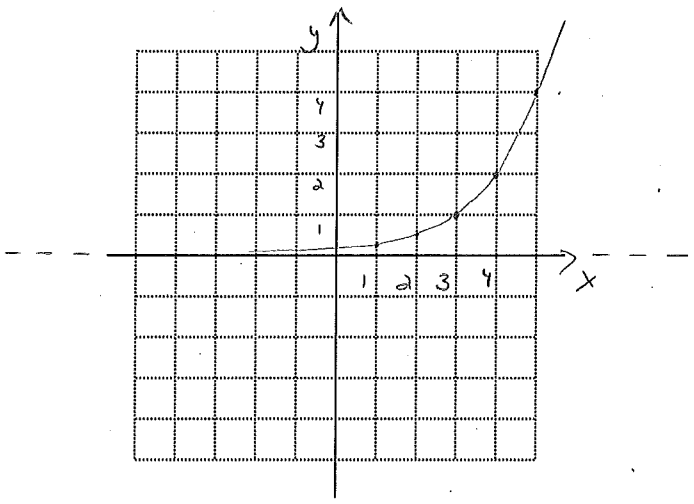
Total: 40 points

1. Sketch the graph of the following function. Include at least two accurate points on your sketch and also indicate the location of any asymptotes. (5 points)

$f(x) = 2^{x-3}$

↑

this is the graph $y = 2^x$ shifted right by 3



asymptote is $y = 0$ (the x-axis)

2. Find each of the following, if possible. Give exact answers. (4 points)

a) $\log_4(-16)$ undefined

b) $\log_4\left(\frac{1}{16}\right) = \log_4 4^{-2}$ -2

c) $\log_2 \sqrt[7]{2} = \log_2 2^{1/7}$ $1/7$

d) $\log_8\left(\frac{1}{2}\right) = \log_8 8^{-1/3}$ $-1/3$

3. Consider the following functions: $f(x)=\frac{1-x}{x}$, $f^{-1}(x)=\frac{1}{x+1}$ (6 points)

a) domain of $f(x)$: $\{x \mid x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$ (1)

range of $f(x)$: $\{x \mid x \neq -1\}$ or $(-\infty, -1) \cup (-1, \infty)$ (1)

b) Use composition of functions to show that the functions are inverses of each other.

$(f \circ f^{-1})(x) = f(f^{-1}(x))$

$= \dfrac{1 - \frac{1}{x+1}}{\frac{1}{x+1}}$

$= \left(1 - \frac{1}{x+1}\right)\left(\frac{x+1}{1}\right)$

$= x + 1 - 1$

$= x$ ✓ (2)

$(f^{-1} \circ f)(x) = \dfrac{1}{\frac{1-x}{x} + 1}$

$= \dfrac{1}{\frac{1-x}{x} + \frac{x}{x}}$

$= \dfrac{1}{\frac{1}{x}}$

$= x$ ✓ (2)

(-1) if mix up $(f \circ f^{-1})$ and $(f^{-1} \circ f)$

4. Find the inverse of the following function. (3 points)

$f(x) = \ln(x+2)$

$y = \ln(x+2)$ replace (½)

$x = \ln(y+2)$ swap (½)

$e^x = y + 2$

$e^x - 2 = y$ } solve (2)

$f^{-1}(x) = e^x - 2$ replace

inverse: $f^{-1}(x) = e^x - 2$

5. Solve. Give exact answers. (12 points)

a) $2^{3x-7} = 32$

$\{4\}$

$2^{3x-7} = 2^5$ (1)

$3x - 7 = 5$ (1)

$3x = 12$

$x = 4$ (1)

(3)

b) $\log(x-7) + \log(x-4) = 1$

$\{9\}$ (1)

$\log(x-7)(x-4) = 1$ (1)

$(x-7)(x-4) = 10^1$ (1)

$x = 9, \cancel{2}$ (1)

extraneous

$x^2 - 11x + 28 = 10$

$x^2 - 11x + 18 = 0$ (1)

$(x-9)(x-2) = 0$

(5)

c) $2^{3x} = e^{x-5}$

$\left\{\frac{5}{1-3\ln 2}\right\}$

$\ln 2^{3x} = \ln e^{x-5}$ (½)

$3x \ln 2 = x - 5$ (½)

$5 = x - 3x \ln 2$ (1)

$5 = x(1 - 3\ln 2)$ (1)

$x = \frac{5}{1-3\ln 2}$ (1)

(4)

(-1) giving exact answer
note: $x \approx -4.63202$

6. Simplify. (4 points)

a) $\ln(x^2-9)-\ln(x-3)$ $= \ln\left(\frac{x^2-9}{x-3}\right)$

$= \ln\frac{(x+3)(x-3)}{(x-3)}$

$\ln(x+3)$

b) $3^{2\log_3 5x}$

$= 3^{\log_3 (5x)^2}$

$= (5x)^2$

$= 25x^2$

$25x^2$

7. You are taking a road trip in a car without air conditioning, and the temperature in the car is 38°C. You buy a cold pop at a gas station. The initial temperature of the pop is 5°C, and after 15 minutes in the car, the pop's temperature is 16°C. What will be the temperature of the pop when it's been in the car for 25 minutes? (6 points)

finding k:

$T = T_0 + (T_1 - T_0)e^{-kt}$

$16 = 38 + (5-38)e^{-k\cdot 15}$

$-22 = -33e^{-15k}$

$\frac{2}{3} = e^{-15k}$

$\ln\frac{2}{3} = -15k$

$k = -\frac{1}{15}\ln\left(\frac{2}{3}\right)$

≈ 0.027031

finding T at 25 min:

$T = T_0 + (T_1 - T_0)e^{-kt}$

$= 38 + (5-38)e^{+\frac{1}{15}\ln(2/3)\cdot 25}$

$= 21.2109\ ^\circ C$

$= 21\ ^\circ C$

After 25 minutes, the pop is at 21°C.

note: after 40 minutes, pop is at 26.8°C (-1)