Math 173 – Quiz #3

February 19, 2015 **Name:** Solution Set

Instructor: Patricia Wrean

Total: 40 points

1. Find the inverse of the following, and state the inverse's domain and range. (5 points)

$f(x)=\dfrac{x}{4+x}$

replace: $y=\dfrac{x}{4+x}$

swap: $x=\dfrac{y}{4+y}$ (1)

solve

$x(4+y)=y$

$4x+xy=y$ (2)

$4x=y-xy$

$4x=y(1-x)$

$y=\dfrac{4x}{1-x}$

$f^{-1}(x)=\dfrac{4x}{1-x}$

$f^{-1}(x)=\dfrac{4x}{1-x}$

domain: $\{x \mid x \neq 1\}$ (1)

range: $\{x \mid x \neq -4\}$ (1)

range of inverse equals domain of function

2. Find each of the following. Give exact answers. (4 points)

a) $\log_4 \dfrac{1}{64} = \log_4 4^{-3} = -3$ -3

b) $\log \sqrt[7]{100} = \log 100^{1/7} = \log 10^{2/7}$ $2/7$

c) $\log_{64} 2 = \log_{64} 64^{1/6}$ $1/6$

d) $\log_3 0$ undefined

3. Calculate the following, rounding your answer to two decimal places. (2 points)

$\log_{1.78} 71.3 = \frac{\ln 71.3}{\ln 1.78} \approx 7.39993$

7.40

4. Simplify. (4 points)

$2\log_2(6k) - \frac{1}{2}\log_2(81k^4)$

2

method #1:

$\log_2 (6k)^2 - \log_2 \sqrt{81k^4}$

$\log_2 (36k^2) - \log_2 (9k^2)$

$\log_2 \frac{36k^2}{9k^2}$

$\log_2 4$

2

method #2

$2\log_2 6 + 2\log_2 k - \frac{1}{2}(\log_2 81 + \log_2 k^4)$

$2\log_2 6 + 2\log_2 k - \frac{1}{2}\log_2 81 - \frac{1}{2}\log_2 k^4$

$2\log_2 6 + 2\log_2 k - \log_2 9 - 2\log_2 k$

$\log_2 36 - \log_2 9$

$\log_2 \frac{36}{9}$

$\log_2 4$

2

5. Sketch the graph of the following function. Include at least two accurate points on the graph and also indicate the location of any asymptotes. (4 points)

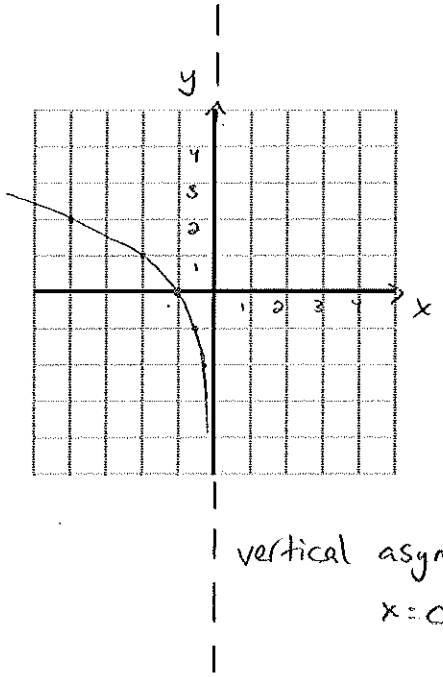
$f(x) = \log_2(-x)$

$y = \log_2 -x$

$2^y = -x$

$x = -2^y$

x	y
-1/8	-3
-1/4	-2
-1/2	-1
-1	0
-2	1
-4	2
-8	3



6. Use composition of functions to show that the following functions are inverses. (4 points)

$f(x)=\frac{1}{2}\log_3(x),\ f^{-1}(x)=3^{2x}$

$(f\circ f^{-1})(x) = f(f^{-1}(x))$

$= \frac{1}{2}\log_3 3^{2x}$

$= \frac{1}{2}(2x)$

$= x$ ✓

$(f^{-1}\circ f)(x) = f^{-1}(f(x))$

$= 3^{2\cdot\frac{1}{2}\log_3 x}$

$= 3^{\log_3 x}$

$= x$ ✓

7. Solve. Give exact answers. (7 points)

a) $8^{2-y}=10^{y+1}$

$\left\{\frac{2\log 8 - 1}{1+\log 8}\right\}$

$\log 8^{2-y} = \log 10^{y+1}$ (1)

$(2-y)\log 8 = y+1$ (1)

$2\log 8 - y\log 8 = y+1$

$2\log 8 - 1 = y + y\log 8$

$= y(1+\log 8)$ (1)

$y = \frac{2\log 8 - 1}{1+\log 8}$ (1)

(4)

b) $\ln(3-x)+\ln(3+x)=\ln x+\ln(1-x)$

$\emptyset$

$\ln(9-x^2) = \ln x(1-x)$

$9-x^2 = x-x^2$

$9 = x$

check: $\ln(6) + \ldots$

$x=9$ is extraneous

(3)

8. An investment is compounded quarterly at 4% interest. How long does it take this investment to double in value? (4 points)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2P = P\left(1 + \frac{0.04}{4}\right)^{4t}$$

$$2 = (1.01)^{4t}$$

$$\ln 2 = \ln 1.01^{4t}$$

$$\ln 2 = 4t \ln 1.01$$

$$t = \frac{\ln 2}{4 \ln 1.01}$$

$$\approx 17.4152$$

$\approx$ 17 years

9. In 1969, Dr. Evil demands one million dollars from the UN, and in 1997, he asks the UN for one hundred billion dollars. Assuming continuous compounding, (6 points)

   a) what is the rate of growth for evil villain extortion demands?

   b) if the next Austin Powers movie comes out in 2017, how much money should Dr. Evil be demanding from the UN then?

a) $A = Pe^{rt}$

$10^{11} = 10^6 e^{r \cdot 28}$

$10^5 = e^{28r}$

$\ln 10^5 = 28r$

$r = \frac{\ln 10^5}{28}$

≈ 0.411176

$\approx 41\%$

(yep, that's evil!)

b) $A = Pe^{rt}$

$= 10^6 e^{0.411176(48)}$

$= 3.72759 \times 10^{14}$

$= 3.7 \times 10^{14}$

(or can use 10^{11} and 20 years to get the same result)

The rate is 41% for evil villian extortion demands, and Dr Evil should demand 370 TRILLION DOLLARS from the UN.