Math 173 – Quiz #4

March 12, 2015 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 40 points

1. Convert the angles in radians to degrees and the angles in degrees to radians. Show your work and leave any answers in radians as multiples of π. (4 points)

   a) $36° = 36° \cdot \left(\frac{\pi}{180°}\right) = \frac{\pi}{5}$ $\pi/5$ (or 0.2π)

   b) $\frac{-5\pi}{9} = -\frac{5\pi}{9}\left(\frac{180°}{\pi}\right) = -100°$ $-100°$

2. Use a calculator to evaluate the following. Round to two decimal places if the answer is approximate. (3 points)

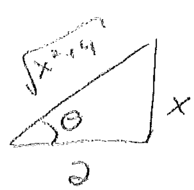
   a) $\sec\left(-\frac{2\pi}{5}\right) = \frac{1}{\cos(-2\pi/5)} \approx 3.23607$ 3.24

   b) $\tan^{-1}(1) = 45°$ or $\frac{\pi}{4}$ or $\underbrace{0.785398}_{\text{rounds to } 0.79}$ $45°$ or $\pi/4$

   c) $\sin(-3) = -0.14112$ -0.14

3. Evaluate. Assume that any angles would be in the first quadrant. (3 points)

   $\sin\left(\tan^{-1}\left(\frac{x}{2}\right)\right)$ – find $\sin\theta$ where $\theta = \tan^{-1}\left(\frac{x}{2}\right)$



$$\sin\theta = \boxed{\frac{x}{\sqrt{x^2+4}}}$$

(1) triangle
(1) $\sqrt{x^2+4}$
(1) $\sin\theta$

4. Use the sum and/or difference identities to simplify the following (4 points)

$\tan(\pi - x)$

method #1:

$$\tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)}$$

$$= \frac{\cancelto{0}{\sin\pi}\cos x - \cancelto{-1}{\cos\pi}\sin x}{\cancelto{-1}{\cos\pi}\cos x + \cancelto{0}{\sin\pi}\sin x}$$

$$= \frac{\sin x}{-\cos x}$$

$$= -\tan x$$

method #2

$$\tan(\pi - x) = \frac{\cancelto{0}{\tan\pi} - \tan x}{1 + \cancelto{0}{\tan\pi}\tan x}$$

$$= \frac{-\tan x}{1}$$

$$= -\tan x$$

5. Solve, finding all solutions in $[0,2\pi)$ or $[0,360^\circ)$. (5 points)

$\cos 2x = 0$ ____________________

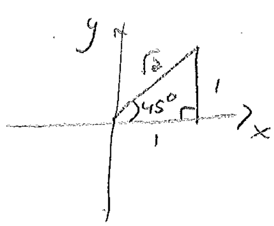
method #1

$$2\cos^2 x - 1 = 0$$

$$2\cos^2 x = 1$$

$$\cos^2 x = \tfrac{1}{2}$$

$$\cos x = \pm\frac{1}{\sqrt{2}}$$



$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

method #2:

$$1 - 2\sin^2 x = 0$$

$$-2\sin^2 x = -1$$

$$\sin^2 x = \tfrac{1}{2}$$

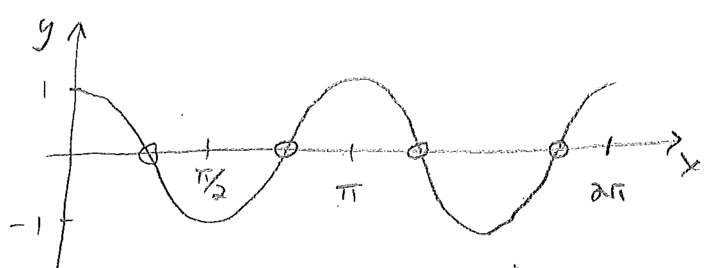
$$\sin x = \pm\frac{1}{\sqrt{2}}$$

same diagram as method #1, and same soln set

method #3

$y = \cos 2x$ has period π:

so 2 periods in 2π



4 solns (as before)

6. Prove the following trig identity. (5 points)

$$\frac{\sin 3x \cos 2x - \cos 3x \sin 2x}{\csc x} = \frac{\tan^2 x}{1+\tan^2 x}$$

(1) $\frac{\sin(3x - 2x)}{\csc x} = \frac{\tan^2 x}{\sec^2 x}$ (1)

(1) $\frac{\sin x}{\csc x} = \tan^2 x \cdot \cos^2 x$

(1) $\sin x \cdot \sin x = \frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x}$ (1)

$\sin^2 x = \sin^2 x$ ✓ QED

7. Simplify. (5 points)

$$\frac{1-\cos 2x}{1+\cos 2x} + \frac{1}{1-\csc^2 x}$$

0

(1) $\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x - 1)} + \frac{1}{-\cot^2 x}$ (1)

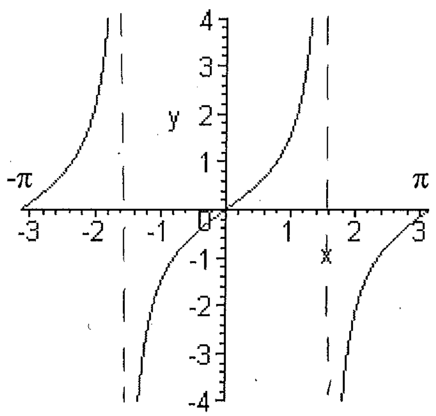
(1) $\frac{\cancel{2}\sin^2 x}{\cancel{2}\cos^2 x} - \frac{1}{\cot^2 x}$

$\tan^2 x - \tan^2 x$ (1)

(1) 0

(4.5) make into single fraction

8. Consider the graph below. State which of the six basic trig functions it is by giving the equation of the graph. Sketch in the positions of any asymptotes. Calculate the function's period and range. Is this function even, odd, or neither? (5 points)



equation: $y = \tan x$

period: π

range: $\mathbb{R}$ or $(-\infty, \infty)$

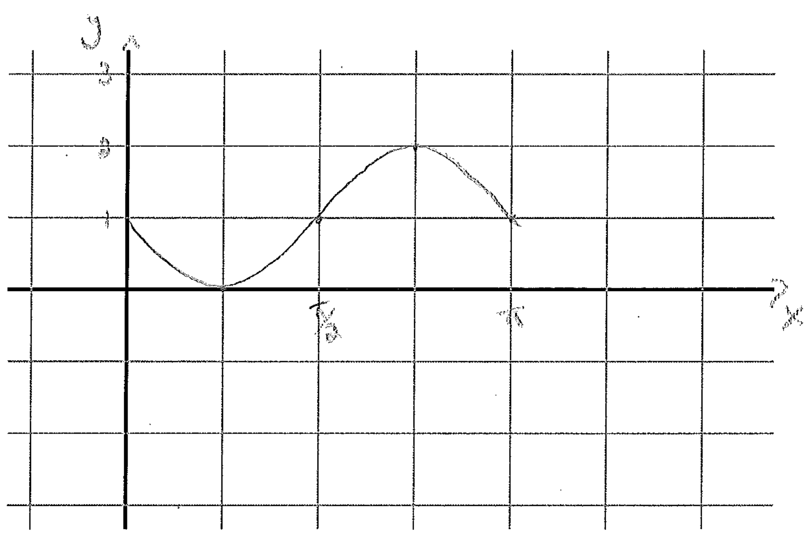
odd

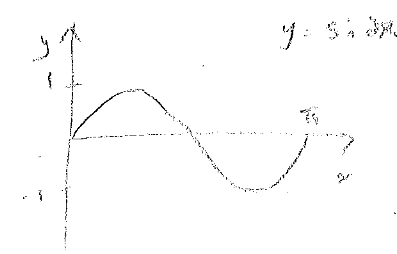
9. Sketch a graph of the function $y = \sin(-2x) + 1$, and state the function's period and amplitude. Include at least one full period in your sketch. 6 (~~5~~ points)

period: π 1

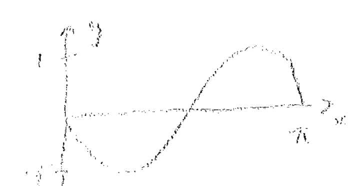
amplitude: 1 1

$$\text{period} = \frac{2\pi}{|B|} = \frac{2\pi}{|-2|} = \pi$$





$y = \sin 2x$



$y = \sin(-2x)$

1 correct shape
1 flipped over axis
1 correct shift
1 correct amplitude