Math 173 – Quiz #5

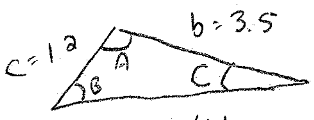
March 19th, 2009 **Name:** Solution Set

Instructor: Patricia Wrean

Total: 40 points

1. Use the information given to solve the following triangles, if possible. Round off all **angles** to the nearest integer. (10 points)

   a) $a = 4.1, b = 3.5, c = 1.2$ $A = 112°, B = 52°, C = 16°$



choosing to solve for A first: (opposite longest side)

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3.5^2 + 1.2^2 - 4.1^2}{2(3.5)(1.2)}$$

$A = 111.8° = 112°$ (3)

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

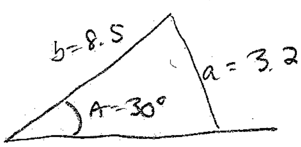
$$\sin B = \frac{b \sin A}{a} = \frac{3.5 \sin 111.8°}{4.1}$$

$B = 52.4281°$
$= 52°$ (2)

$C = 180° - A - B$
$= 16°$ (2)

[note: if calculate C to 16° first and round off, will get $A = 110°$ and $B = 54°$ instead]

   b) $a = 3.2, b = 8.5, A = 30°$ no such triangle (2)



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

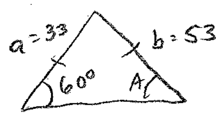
$$\sin B = \frac{b \sin A}{a} = \frac{8.5 \sin 30°}{3.2}$$

$= 1.328$

$\sin B > 1$, so B does not exist (1)

2. For the following triangle, calculate the length of side c. (2 points)

a) $a = 53$, $b = 53$, $B = 60°$ $\underline{c = 53}$



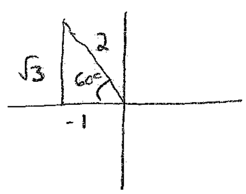
so $A = 60°$ (isosceles)

$C = 60°$ $(180° - A - B)$

Δ is equilateral

3. Write the following complex number in the form $a+bi$. (2 points)

$2(\cos 120° + i \sin 120°)$ $\underline{-1 + i\sqrt{3}}$



4. Write the following either in trig form or as $re^{i\theta}$, your choice. (3 points)

$-7i$ $\underline{7e^{i\,3\pi/2}}$ (or coterminal)

$7e^{i\,270°}$

5. Multiply the two numbers below using whatever method you wish. (3 points)

$$3\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) \times 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

$\underline{12e^{i\pi/2}}$

or $12i$

$3e^{i(-\pi/3)} \times 4e^{i(5\pi/6)}$

$12e^{i(-\pi/3 + 5\pi/6)}$

$12e^{i(-2\pi/6 + 5\pi/6)}$

$12e^{i\,3\pi/6}$

$12e^{i\pi/2}$

6. Calculate the coordinates of the vertex and focus of the following parabola. Also, state the equations of the directrix and axis of symmetry. You do not need to sketch the parabola, though I've included a grid for any rough work. (5 points)

$(x+3)^2 = 8(y-1)$

$(x-h)^2 = 4p(y-k)^2$

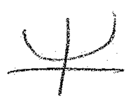
so $4p = 8$

$p = 2$

$(h,k) = (-3,1)$

and since it's x^2 and y

and p is $+$



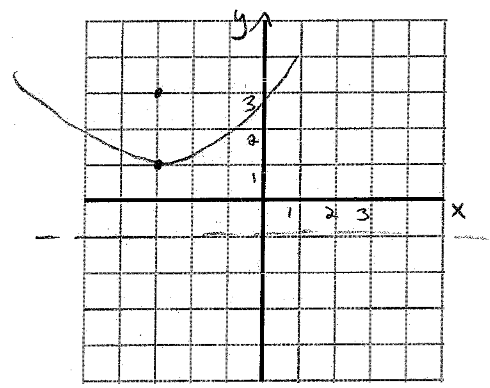
orientation

vertex: $(-3, 1)$ 1

focus: $(-3, 3)$ 1

directrix: $y = -1$ 1

axis of symmetry: $x = -3$ 1



7. Give the equation of the ellipse with vertices at $(3, \pm 2)$ and foci at $(3, \pm \sqrt{3})$. Also, sketch the graph as accurately as possible. (5 points)

$a = 2$

$c = \sqrt{3}$

$a^2 = b^2 + c^2$

$b^2 = a^2 - c^2$

$= 4 - 3$

$= 1$

$b = 1$

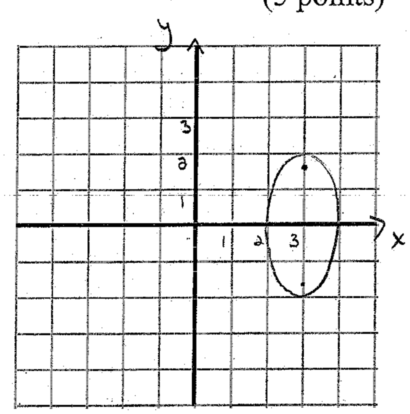
1

$(h, k) = (3, 0)$

vertically oriented, so

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

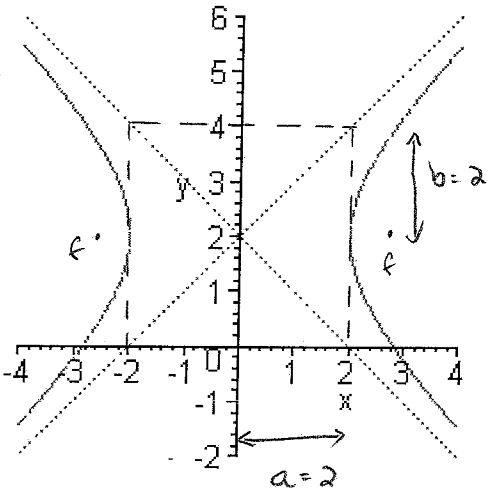
$\frac{(x-3)^2}{1} + \frac{(y-0)}{4} = 1$



2

equation: $\frac{(x-3)^2}{1} + \frac{y^2}{4} = 1$ 2

8. Write the equation for the hyperbola shown in the graph below. (The dashed lines in the graph are the asymptotes.) Also, give the coordinates of the foci. (5 points)



equation: $\frac{x^2}{4} - \frac{(y-2)^2}{4} = 1$ (2)

foci: $(\pm 2\sqrt{2}, 2)$ (1)

$(h, k) = (0, 2)$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{x^2}{4} - \frac{(y-2)^2}{4} = 1$

$c^2 = a^2 + b^2$

$c = \sqrt{8} = 2\sqrt{2}$

a, b, c } (2)

9. Complete the square to convert the following equation to the form of a conic section. Which conic section is it (parabola, circle, ellipse, hyperbola)? What are the coordinates of the centre? (5 points)

$x^2 + y^2 - 12x + 6y + 29 = 0$

$x^2 - 12x + y^2 + 6x = -29$

$x^2 - 12x + \underline{36} + y^2 + 6x + \underline{9} = -29 + 36 + 9$

$(x-6)^2 + (y+3)^2 = 16$

(2)

equation: $(x-6)^2 + (y+3)^2 = 4^2$ (1)

which conic? circle (1)

centre: $(6, -3)$ (1)