3 Probability

In statistics, an *experiment* is any procedure than can be repeated and has a well-defined set of possible outcomes. An experiment can be something as simple as rolling a die and noting the number, or something as complicated as finding the mass of an electron.

> **Definition:** The sample space of an experiment is the set of all possible outcomes. An event is a subset of a sample space.
> **Notation:** A sample space is usually denoted by S.
> If E is an event, then $n(E)$ denotes the number of outcomes in E.

Example 3.1. An experiment consists of flipping a coin 3 times and noting if it's heads (H) or tails (T).

(a) Find the sample space.

(b) Let E be the event that exactly two heads occur. Find $n(E)$.

3.1 Probabilities Involving Equally Likely Outcomes

There are a number of ways to define the probability of an event. In this section we consider the simplest case.

> **Definition:** If each outcome of an experiment is *equally likely* then the probability of an event E is the ratio
> $$P(E) = \frac{n(E)}{n(S)},$$
> where S is the sample space of the experiment.

Since E is a subset of S, we always have $0 \leq P(E) \leq 1$. For a finite sample space, $P(E) = 0$ if and only if $E = \varnothing$; and $P(E) = 1$ if and only if $E = S$.

Example 3.2. An experiment consists of flipping a coin 3 times and noting if it's heads (H) or tails (T). What is the probability of getting exactly two heads?

Example 3.3. An experiment consists of randomly selecting a number between 1 and 40 (inclusive). Find the probability of getting a multiple of 5 or 7.

Example 3.4. An experiment consists of rolling a pair of standard 6-sided dice and noting the number. Find the probability of getting a sum of at most 5.

Example 3.5. Four sidewalk squares have the following number of cracks: 4, 6, 7, 9. Pick two of the squares at random (order doesn't matter). Find the probability that they have at least 15 cracks in total.

3.2 Venn Diagrams

Demonstration: Coffee and Spicy Food

	like coffee	don't like coffee
like spicy food		
don't like spicy food		

(a) Represent this data in a Venn diagram.

(b) Find the probability that a randomly chosen student likes coffee.

(c) Find the probability that a randomly chosen student likes coffee but doesn't like spicy food.

(d) Find the probability that a randomly chosen student likes coffee or spicy food.

Example 3.6.

In a class of 45 students, 26 have jobs and 17 have cars. Of those who don't have a car, 10 have jobs. Find the probability that a student has:

(a) a car or a job.

(b) a car but not a job.

Example 3.7. On any given day, the probability that Machine I breaks down is 4%, the probability that Machine II breaks down is 7%, and the probability that both machines break down is 2%. Find the probability that Machine II breaks down and Machine I doesn't.

3.3 Probability Rules

- $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

- $n(\text{not } A) = n(S) - n(A)$

  $P(\text{not } A) = 1 - P(A)$

- If Experiment 1 has n possible outcomes and Experiment 2 has m possible outcomes, then the experiment consisting of Experiment 1 followed by Experiment 2 has

  $$n \cdot m$$

  possible outcomes.

Example 3.8. Find the number of possible outcomes when a standard 6-sided die is rolled

(a) twice.

(b) 5 times.

Example 3.9. A password consists of 7 digits, each chosen from 0,1,2, $\cdots$, 9. Find the

(a) total number of passwords possible.

(b) number of passwords that end with 3.

(c) number of passwords that don't end with 3.

(d) the probability that a password starts with 4.

(e) the probability that a password doesn't start with 4.

(f) the probability that a password doesn't start with 32.

(g) the probability that a password contains a least one 4.

(h) the probability that a password starts with 29 or ends with 1.

Additional Notes

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