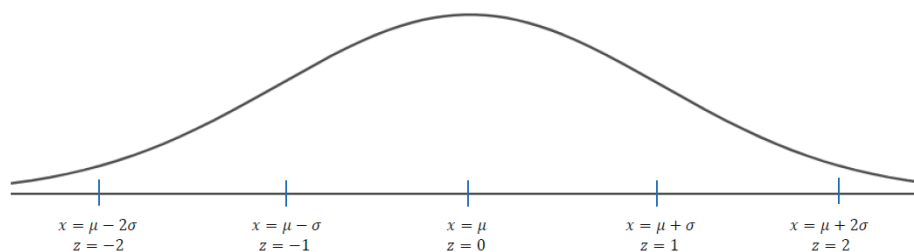


7 The Normal Distribution

The most important and most widely used distribution in statistics is the normal distribution.

Definition: A continuous random variable, X , with mean μ and SD σ has a normal distribution if its p.d.f. is

$$f(x) = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}.$$



By use of a change of variable called the z-score,

$$z = \frac{x - \mu}{\sigma},$$

we can standardize any normal distribution. The standard normal distribution has the p.d.f.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}.$$

The graph of a normal distribution is often referred to as a *bell curve*. Since the p.d.f. is very difficult to integrate, we use the *Standard Normal Distribution Table* to find the area under the bell curve corresponding to

$$\int_a^b f(x) dx = P(a \leq X \leq b) = P\left(\frac{a - \mu}{\sigma} \leq z \leq \frac{b - \mu}{\sigma}\right).$$

Example 7.1. Find the proportion of x -values that are within one standard deviation of the mean for a normal distribution; that is, find $P(\mu - \sigma \leq X \leq \mu + \sigma)$.

Example 7.2. The volume in bottles of gingerale is normally distributed with a mean of 2.01 L and a SD of 0.13 L. Find the probability that a bottle has a volume

(a) between 1.77 and 2.29 L.

(b) more than 1.91 L.

Example 7.3. [1, p. 45] Mopeds are popular in Europe because of their mobility, ease of operation, and low cost. A 2008 article described a rolling bench test for determining maximum vehicle speed. A normal distribution with $\mu = 46.8$ km/h and $\sigma = 1.75$ km/h is proposed.

(a) What proportion of mopeds have a maximum speed that is at most 50 km/h?

(b) What proportion of mopeds have a maximum speed that is at least 48 km/h?

(c) What speed separates the fastest 75% of all mopeds from the others?

Example 7.4. [1, p. 45] Suppose the flow of current (in milliamps) in wire strips of a certain type under specified conditions can be modeled with a normal distribution having $\mu = 20$ and $\sigma = 1$. How large must a current flow be to be among the largest 5% of all flows?

Example 7.5. [1, p. 41] The time that it takes a driver to react to the brake on a decelerating vehicle is critical in avoiding rear-end collisions. A 1993 article from *Ergonomics* suggests that reaction time for an in-traffic response to a brake signal from standard brake lights can be modeled with a normal distribution having parameters $\mu = 1.25$ sec and $\sigma = 0.46$ sec. In the long run, what proportion of reaction times will

(a) be between 1.00 sec and 1.75 sec?

(b) exceed 2 sec?

Example 7.6. The time it takes to inspect a certain type of production component is normally distributed with a mean of 6.8 s. Find the SD of the inspection times if 26.62% of inspection times are between 6.2 and 7.4 s.

Additional Notes

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